OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2638

Mechanics 2

Friday **21 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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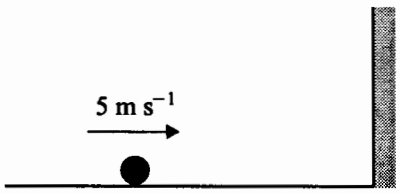
1



A small sphere of mass 0.2 kg is free to move on a smooth horizontal surface. The sphere is projected directly towards a vertical wall with a speed of 5 m s^{-1} (see diagram). The coefficient of restitution between the sphere and the wall is $\frac{1}{2}$.

(i) Write down the speed of the sphere after the impact with the wall. [1]

(ii) Find the magnitude and direction of the impulse which the wall exerts on the sphere. [3]

2

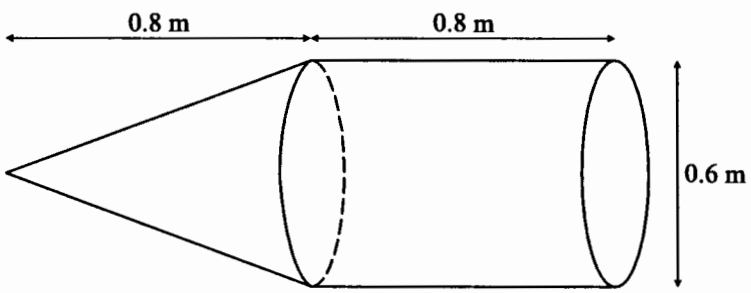


Fig. 1

A uniform solid cone has mass 0.5 kg, height 0.8 m and base diameter 0.6 m. A uniform solid cylinder has mass 0.7 kg, length 0.8 m and diameter 0.6 m. The cone is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

(i) Show that the distance of the centre of mass of the combined object from the vertex of the cone is 0.95 m. [4]

(ii)

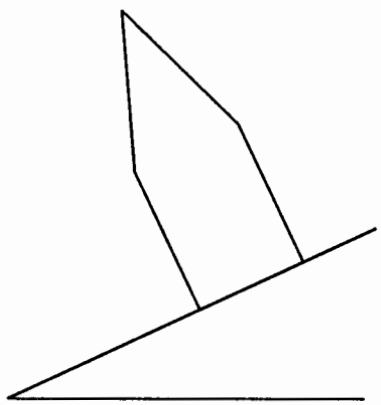


Fig. 2

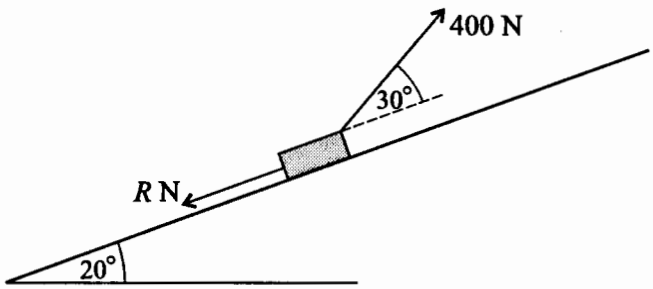
The combined object is placed on an inclined plane (see Fig. 2 for a diagram of the cross-section). The surface of the plane is rough enough to prevent slipping. Given that the object is about to topple, calculate the angle which the plane makes with the horizontal. [2]

3 A motorcycle and its rider have a total mass of 350 kg. The maximum power of the motorcycle is 20 kW and its maximum speed on a horizontal road is 40 m s^{-1}. The total resistance to motion of the motorcycle and rider has magnitude kv^2 newtons, where v m s^{-1} is the speed of the motorcycle.

(i) Show that $k = \frac{5}{16}$. [3]

(ii) Find the acceleration of the motorcycle when it is moving at 30 m s^{-1} on a horizontal road, with the engine working at its maximum rate. [3]

4

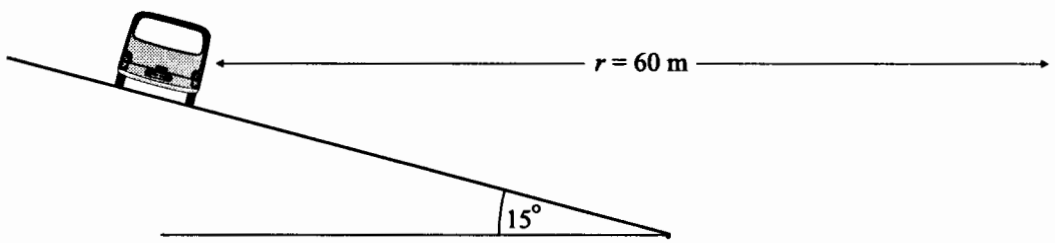


A box of mass 50 kg is dragged up a slope by a man who exerts a constant force of magnitude 400 N at an angle of 30° above the slope. The slope is inclined at 20° to the horizontal and the total resistance to motion of the box is R N (see diagram). The box is dragged from rest at a point O and passes a point P, 25 m from O, with a speed of 2 m s^{-1}.

(i) Calculate the work done by the 400 N force in dragging the box from O to P. [2]

(ii) By considering the increase in kinetic and potential energies of the box and the work done by the 400 N force, find the value of R. [5]

5



A car of mass 500 kg moves in a horizontal circular path of radius 60 m around a bend that is banked at an angle of 15° to the horizontal (see diagram).

(i) In a model of this motion the surface of the road is smooth. The car does not slip up or down the slope.

(a) Find the magnitude of the normal reaction force exerted on the car by the road. [2]

(b) Calculate the speed of the car. [3]

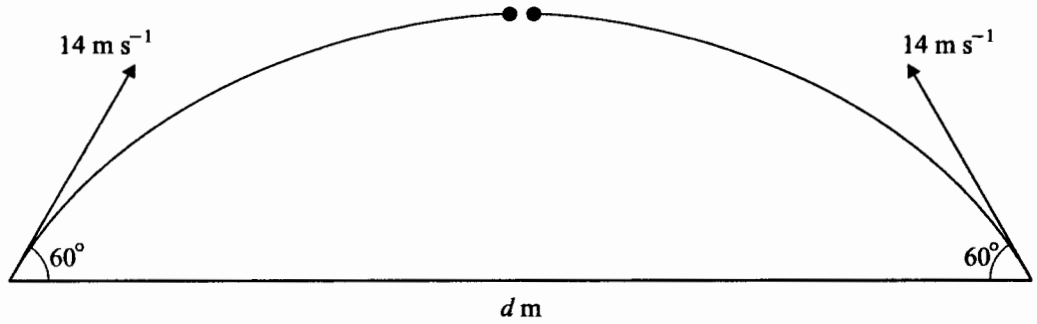
(ii) In an improved model, the surface of the road is rough and the coefficient of friction is $\frac{3}{7}$. The car travels round the bend as fast as possible without slipping up the slope.

(a) Find the magnitude of the normal reaction force exerted on the car by the road. [3]

(b) Calculate the speed of the car. [3]

[Turn over

6



Two small spheres are projected simultaneously from points d metres apart on horizontal ground. Each sphere has an initial velocity of 14 m s^{-1} at an angle of 60° above the horizontal. The spheres move freely under gravity in the same vertical plane and collide with each other at the instant when they are travelling horizontally (see diagram).

(i) Calculate the height above the ground of the point of collision. [2]

(ii) Find d. [4]

The spheres have equal mass and the coefficient of restitution between them is $\frac{4}{7}$.

(iii) Calculate the speed and direction of motion of one of the spheres when it hits the ground. [7]

7 A uniform ladder of mass 20 kg and length 5 m rests against a smooth vertical wall with its lower end resting on rough horizontal ground. The coefficient of friction between the ladder and the ground is $\frac{2}{5}$. The ladder is inclined at 60° to the horizontal.

(i) Find the normal reaction forces at the ground and the wall. [4]

(ii) Find the frictional force at the ground. Hence show that the ladder is not in danger of slipping. [2]

A man of mass 80 kg steps on to the bottom of the ladder.

(iii) Find how far the man can climb up the ladder before it slips. [7]

Mark scheme for M2 2638 (January 2005) (final)

1	(i)	$v = 2½$	B1	1	+ only	
	(ii)	$I = 0.2x2½ - (- 0.2x5)$	M1		Impulse = change of momentum	
		$I = 1.5$ Ns	A1		+ only	
		←	B1	3	to the left	4

2	(i)	c.of m. of cone 0.6 or 0.2	B1		clearly from vertex (0.6) or base(0.2)	
		moments	M1		moments about vertex or about base	
		$1.2d = 0.5x0.6 + 0.7x1.2$	A1		$1.2e = 0.7x\ 0.4 + 0.5x1.0$ ($e = 0.65$)	
		$d = 0.95$	A1	4	**AG**	
	(ii)	$\tan\theta = 0.3/0.65$	M1		geometry must be correct	
		$\theta = 24.8°$	A1	2		6

3	(i)	$D = 500$ N	B1			
		$500 = k \times 40^2$	M1			
		$k = 5/16$	A1	3	**AG** (or 0.3125)	
	(ii)	$20{,}000/30 - 5/16x30^2 = 350a$	M1		must have attempt at all 3 parts but not 500 for driving force or R	
		"	A1			
		$a = 1.10$ ms^{-2}	A1	3	or 1.1 or 185/168 or 1 + 17/168	6

4	(i)	$400\cos30° \times 25$	M1			
		8660 Nm	A1	2	or 8.66 k…	
	(ii)	inc.in K.E. = 100	B1		$½.50.2^2$	
		inc in P.E. = 4190	B1		$50x9.8x25\sin20°$	
		$100+4190+25R = 8660$	M1		must have 4 terms and 25R	
		"	A1			
		$R = 175$	A1	5		7
	or	$a = 0.08$	B1		no P.E. & K.E. 3 marks available only if N II used	
		$400\cos30°-R-50g\sin20°=50a$	M1			
		$R = 175$	A1			

5	(i)(a)	$R\cos15° = 500x9.8$	M1		for resolving vertically	
		$R = 5070$ N	A1	2	components of R needed for M1s	
	(b)	$R\sin15° = 500v^2/60$	M1		N II and $a = v^2/r$ (not for R=500g)	
		"	A1		R can be x for this (value needed)	
		$v = 12.6$ ms^{-1}	A1	3		
	(ii)(a)	$R\cos15°=500.9.8+F\sin15°$	M1		for resolving vertically (3 parts) unknown F and R (M1 & B1)	
		$F = 3/7\ R$	B1			
		$R = 5730$ N	A1	3		
	(b)	$R\sin15°+F\cos15°=500.v^2/60$	M1		N II & $a = v^2/r$ (3 parts) (accept R=500g for M1 only)	
		"	A1		F & R can be wrong for 1st A1	
		$v = 21.5$ ms^{-1}	A1	3		11

6	(i)	$0 = 14^2 \sin^2 60° - 2.9.8h$	M1		or use of $v^2 \sin^2\theta / 2g$	
		$h = 7.5$ m	A1	2		
	(ii)	$0 = 14\sin 60° - 9.8t$	M1		use of $v = u + at$ or similar	
		$t = 1.24$ s (t to max height)	B1		or $5/7\sqrt{3}$ or total time 2.47	
		$d = 2x14\cos 60°x1.24$	M1		must have 2	
		$d = 17.3$	A1	4	range=$u^2\sin 2\theta/g$ = 17.3 scores 4	
	(iii)	$v = 4/7 \times 7$	M1			
		$v = 4$	A1		±	
		vert. comp $v = 12.1$	B1		or $14\sin 60°$ or $\sqrt{147}$	
		speed=$\sqrt{(4^2 + 12.1^2)}$	M1		may be implied	
		speed = 12.8 ms^{-1}	A1			
		$\theta = \tan^{-1}(12.1/4)$	M1		or equivalent (may be implied)	
		$\theta = 71.7°$ to horizontal	A1	7	or 18.3° to vertical	**13**

7	(i)	R = 196 N (not in terms of g)	B1		reaction force at ground	
		N.5sin60°=20.9.8.2.5cos60°	M1		moments about B or about T	
		"	A1		20.9.8.2½.½+F.5sin60°=R.2½	
		N = 56.6 (not in terms of g)	A1	4	reaction force at wall	
	(ii)	F = 56.6	B1√		√ their N (value required)	
		56.6 < 2/5 x 196 (78.4)	B1	2	or F/R = 0.289 < 0.4	
	(iii)	R' = 980 or 100g	B1			
		F' = 2/5R' = 392 or 40g	B1√		√ their R' (not 196)	
		N' = F'	B1		not 56.6 or 78.4	
		N'.5sin60° =	M1*		M(B) 3 parts (all moments)	
		20.9.8.2.5.½ +80.9.8.dcos60°	A1		any N if substituted	
		solve for d	M1*		depends on previous M1	
		d = 3.71 m	A1	7		**13**
	or	80gecos60°+20g.2.5cos60° +	M1*		M(T) last 5 marks above become	
		F'.5.sin60° = R'.5cos60°	A1		e = dist from T	
		e= 1.29	A1			
		d = 5 - e	M1*			
		d = 3.71	A1√			
	or	80gfcos60°+R'.2.5cos60° =	M1		M(G) last 4 marks become	
		N'.2.5sin60°+F'.2.5sin60°	A1		f = dist from G	
		f = 1.21	A1			
		d = 3.71	A1			